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Variance Estimation in Time Series Regression Models

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The effect of variance estimation of regression coefficients when disturbances are serially correlated in time series regression models is studied. Variance estimation enters into confidence interval estimation, hypotheses testing, spectrum estimation, and expressions for the estimated standard error of prediction. Using computer simulations, the robustness of various estimators, including Estimated Generalized Least Squares (EGLS) was considered. The estimates of variance of the coefficient estimators produced by computer packages were considered. Models were generated with a second order auto-correlated error structure, considering the robustness of estimators based upon misspecified order. Ordinary Least Squares (OLS) (order zero) estimates outperformed first order EGLS. A full comparison of order zero and four estimators indicate that over specification is preferable to under specification.

Key words: Autoregressive models, auto-correlated, disturbances, ordinary least squares, generalized least squares.

Introduction

In the standard linear regression model,

$$y = X\,\beta + u\,, \qquad (1)$$

where y is the $(T \times 1)$ response variable; X is an $(T \times k)$ model matrix; β is a $(k \times 1)$ vector of unknown regression parameters; and u is a $(T \times 1)$ random vector of disturbances, it is well known that Ordinary Least Squares (OLS) yield unbiased, but inefficient estimates for the regression parameters with serially correlated disturbance structures. OLS regression estimates have larger sampling variances than the Generalized Least Squares (GLS) estimator which accounts for auto-correlated nature of disturbances.

An important consideration is the estimation of the standard errors of the estimators, because estimates of the variance enter into usual inference procedures such as prediction and confidence intervals, hypotheses testing, spectrum estimation, expressions for the estimated standard error of prediction, and other inferential procedures.

In practice, if using a statistical package to compute the OLS estimators the variance estimate produced would be based on $\sigma_u^2 \left(X'X\right)^{-1}$, which may be biased for the true variance $\sigma_u^2 \left(X'X\right)^{-1} X'\Sigma X \left(X'X\right)^{-1}$. For GLS estimation ($\Sigma$ known), on the other hand, the variance estimate is unbiased for the true variance of the GLS estimator. It is unclear, however, how the variance estimators for EGLS estimation behave. In order to investigate how well the variance estimators function in the different cases, the ratio of the variance of the OLS estimated variance to that for the estimated GLS estimators from the simulation results was computed.

The most commonly assumed process in both theoretical and empirical studies is the first-order autoregressive process, or AR(1), which can be represented in the autoregressive form as

$$u_t = \rho u_{t-1} + \varepsilon_t\,, \quad \varepsilon_t \sim \text{i.i.d. } N\left(0, \sigma_\varepsilon^2\right) \qquad (2)$$

where ρ is the first order autoregressive disturbance parameter. The second-order autoregressive process, or AR(2) error process, may be written

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$$u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \varepsilon_t \qquad (3)$$

where ϕ_1 and ϕ_2 are the second-order autoregressive disturbance parameters.

Numerous articles describe the efficiency of the OLS coefficient estimator relative to the GLS estimator which takes this correlation into account. Safi & White (2006) have shown that, if the error structure is autoregressive and the dependent variable is non-stochastic and linear or quadratic, the OLS estimator performs nearly as well as its competitors. When faced with an unknown error structure, however, AR(4) may offer the best choice. Koreisha & Fang (2004) investigated the impact that the EIGLS correction may have on forecast performance. They found that, for predictive purposes, not much is gained in trying to identify the actual order and form of the auto-correlated disturbances or in using more complicated estimation methods such as GLS or MLE procedures which often require inversion of large matrices. Krämer & Marmol (2002) showed that OLS and GLS are asymptotically equivalent in the linear regression model with AR(p) disturbance and a wide range of trending independent variables, and that OLS based statistical inference is still meaningful after proper adjustment of the test statistics. Grenander & Rosenblatt (1957) gave necessary and sufficient conditions for X such that the OLS and GLS estimators have the same asymptotic covariance matrix. This class of X matrices includes polynomial and trigonometric polynomial functions of time.

In addition, it is known from Anderson's (1948) results that if the columns of observations on k independent variables are linearly dependent on a set of k eigen vectors of the variance matrix of the errors, then the efficiency of the OLS estimator will be identical with the GLS estimator for most values of the autocorrelation coefficient $|\rho| < 1$. By contrast, if this matrix is allowed to vary arbitrarily, the efficiency of the OLS relative to the GLS estimator with a known autocorrelation coefficient can approach zero. Good references of techniques for analysis in time series models are Anderson (1971) and Fuller (1996).

The GLS estimator based on an under parameterized AR(1) disturbance model structure with an estimated AR(1) coefficient denoted, EIGLS-AR(1) will have the highest variance estimation among the other estimators. For example, for some cases the variance estimation of EIGLS-AR(1) is at least more than six times higher than the OLS estimator. This indicates that EIGLS-AR(1) can be much less efficient than OLS.

This article is organized as follows: Simulation setup, definitions of the mean squared error of the variance for each of the regression coefficients, the bias and the variance of the estimated variance, and the ratio of the variance of the OLS estimated variance to that of four GLS estimators are introduced. Complete simulation results based on the variance of OLS and GLS estimated variance of each of the regression coefficients are shown and the ratio of variance estimation of OLS to that of GLS estimators for each of the regression coefficients is discussed. This simulation study was designed to compare the performance of different estimators and to characterize the effect of the design on the efficiency of OLS. Lastly, conclusions based on the comparison of the variance estimation of OLS and GLS on the regression coefficients is provided.

Methodology

The robustness of various estimators, including estimated generalized least squares (EGLS) was considered. These simulations examined the sensitivity of estimators to model misspecification.

The the ratios of the variances of the OLS estimator relative to four GLS estimated variances were compared: the GLS based on the correct disturbance model structure and known AR(2) coefficients denoted as GLS-AR(2); the GLS based on the correct disturbance model structure but with estimated AR(2) coefficients denoted as EGLS-AR(2); the GLS based on an under parameterized AR(1) disturbance model structure with an estimated AR(1) coefficient denoted as EIGLS-AR(1), and the GLS based on over parameterized AR(4) disturbance model structure with estimated AR(4) coefficients denoted as EIGLS-AR(4). AR(p) GLS corrections disturbances.

Three finite sample sizes (50, 100, and 200) and three non-stochastic design vectors of the independent variable were used; linear, quadratic, and exponential. A standard normal stochastic design vector of length 1,000 was generated, assuming the variance of the error term in AR(2) process was $\left(\sigma_{\varepsilon}^{2}=1\right)$. In addition, 1,000 observations were generated for each of the AR(2) error disturbances with four pairs of autoregressive coefficients: (.2,-.9), (.8,-.9), (.2,-.7), and (.2,-.1).

The regression coefficients β_0, and β_1 for an intercept and the slope were each chosen to equal one. Breusch (1980) has shown that for a fixed design, the distribution of $\dfrac{\hat{\beta}_{EGLS}-\beta}{\sigma_u^2}$ does not depend on the choice for β and σ_u^2, and the result holds even if the covariance matrix Σ is misspecified.

Definition 1
The simulation mean squared error ($\hat{\eta}_{\beta_j}$) of an estimated variance W, of the true variance (τ), is the function defined by $E_{\tau}\left(W-\tau\right)^{2}$. That is

$$\hat{\eta}_{\beta_j}=k^{-1}\sum_{i=1}^{k}\left(W_{ij}-\tau\right)^{2} \tag{4}$$

where j = 0,1, k is the number of simulations,

$$W_{ij}=\widehat{\operatorname{Var}}\left(\beta_{ij}\right),\tau=\operatorname{Var}_T\left(\beta_j\right).$$

An estimate with the smallest value in (4) indicates that it was the most efficient among other estimates.

Definition 2
The bias of an estimated variance (W), of the true variance (τ), is the difference between the expected value of W and τ. That is,

$$\hat{\delta}_{\beta_j}=E_{\tau}W-\tau \tag{5}$$

where

$$E_{\tau}W=k^{-1}\sum_{i=1}^{k}\widehat{\operatorname{Var}}\left(\beta_{ij}\right).$$

An estimator whose bias is identically (in τ) equal to zero is called unbiased and satisfies $E_{\tau}W=\tau$ for all τ.

Note that $\tau=\operatorname{Var}_T\left(\beta_j\right)$ is different for each case of the estimation procedure; since no known explicit formula exists for EGLS cases, this quantity is estimated from the simulation results in all cases.

Definition 3
The variance of the estimated variance (W), of the true variance (τ), is the difference between the estimated mean squared error ($\hat{\eta}_{\beta_j}$), and the bias of an estimated variance W, $\hat{\delta}_{\beta_j}$. That is,

$$\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_j}\right)=\hat{\eta}_{\beta_j}-\hat{\delta}_{\beta_j}^{2} \tag{6}$$

Definition 4
The ratio of the variance of the OLS estimated variance to that of GLS is

$$R_{\beta_j}=\frac{V_j}{V_{ji}} \tag{7}$$

where

$$V_j=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.OLS}}\right),$$
$$V_{ji}=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.GLS}}\right),$$
$$j=0,1,i=1,2,3,4$$

for four GLS estimates such that:

$$V_{j1}=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.GLS-AR(2)}}\right),$$
$$V_{j2}=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.EGLS-AR(2)}}\right),$$
$$V_{j3}=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.EIGLS-AR(1)}}\right),$$
$$V_{j4}=\operatorname{Var}\left(\widehat{\operatorname{Var}}_{\beta_{j.EIGLS-AR(4)}}\right).$$

A ratio (R_{β_j}), less than one indicates that the OLS estimate is more efficient than GLS, if R_{β_j} is close to one then the OLS estimate is nearly as efficient as GLS, otherwise, OLS performs poorly.

S-plus code was written to compute the ratio of the variance of the OLS estimated variance to that of GLS in (7) using the OLS and four GLS estimators.

Results

The simulation results based on the variances of OLS and GLS estimated variance of each of the regression coefficients using four GLS and OLS estimates are now discussed.

Tables (1) and (2) show the simulation results of the variances of OLS and four GLS estimated variance, $\mathrm{Var}\left(\widehat{\mathrm{Var}}_{\beta_0}\right)$ and $\mathrm{Var}\left(\widehat{\mathrm{Var}}_{\beta_1}\right)$ in (6), when the serially correlated disturbance is AR(2) process, under parameterized AR(1), and over parameterized AR(4) for linear design with all selected AR(2) coefficients and all sample sizes.

First, regardless of sample size, the selected autoregressive coefficients for all non-stochastic designs, OLS was more efficient than EIGLS-AR(1) in estimating both β_0 and β_1. This is shown in Table (1), when Φ = (.8,-.9) for a linear design with T=100, $\left[V_0, V_{03}\right]$= [7.9340E-04, 5.8309E-03] and $\left[V_1, V_{13}\right]$ = [8.0950E-04, 5.5626E-03]. For all cases EIGLS-AR(1) was the least efficient estimator. For example, when Φ = (.2,-.9) with T=200, V_{03} = 1.0822E-04 and V_{13} = 1.0868E-04.

Second, regardless of sample size and selected non-stochastic design, OLS was more efficient than GLS in estimating $\left(\beta_0, \beta_1\right)$ with Φ = (.2,-.1). For example, as shown in Table (2), with T=50, $\left[V_0, V_{01}\right]$ = [1.9062E-05, 2.5782E-05] and $\left[V_1, V_{11}\right]$= [1.9848E-05, 2.6844E-05]. Otherwise, the OLS estimator performed less efficiently than the GLS estimator. Furthermore, if Φ = (.2,-.1), OLS was more efficient than GLS estimates; EGLS- AR(2), and EIGLS-AR(4), for

Table 1: Panel (A) - Variances of OLS and GLS Estimators for Linear Design

		(Φ_1, Φ_2) = (.2, -.9)		(Φ_1, Φ_2) = (.8, -.9)	
Size	Estimator	V0	V1	V0	V1
50	VOLS	3.8994E-03	4.0602E-03	6.0748E-03	6.3253E-03
	VGLS AR(2)	1.9922E-06	2.4186E-06	1.3186E-05	1.5923E-05
	VEGLS AR(2)	2.9702E-06	3.5411E-06	2.0197E-05	2.3851E-05
	VEIGLS AR(1)	7.5176E-03	7.5862E-03	5.3587E-02	4.8020E-02
	VEIGLS AR(4)	2.1458E-05	1.9951E-05	1.2105E-04	1.1500E-04
100	VOLS	5.0757E-04	5.1788E-04	7.9340E-04	8.0950E-04
	VGLS AR(2)	2.3634E-07	2.6019E-07	1.5304E-06	1.6804E-06
	VEGLS AR(2)	3.0145E-07	3.2972E-07	2.2467E-06	2.4441E-06
	VEIGLS AR(1)	8.6461E-04	8.7130E-04	5.8309E-03	5.5626E-03
	VEIGLS AR(4)	1.6742E-06	1.7054E-06	9.0592E-06	9.2609E-06
200	VOLS	6.5781E-05	6.6444E-05	9.7015E-05	9.7993E-05
	VGLS AR(2)	3.2956E-08	3.4571E-08	1.5183E-07	1.5907E-07
	VEGLS AR(2)	4.0323E-08	4.2192E-08	2.3086E-07	2.4091E-07
	VEIGLS AR(1)	1.0822E-04	1.0868E-04	6.6333E-04	6.4879E-04
	VEIGLS AR(4)	1.7871E-07	1.8186E-07	1.0469E-06	1.0632E-06

all sample sizes for all design vectors. This is shown in Table (2). For a linear design with sample size T=100; the variances of the estimated variance of (β_0, β_1) using OLS, EGLS-AR(2) and EIGLS-AR(4) were $[V_0, V_{02}, V_{04}]$ = [2.4432E-06, 1.9476E-05, 4.2741E-05] and $[V_1, V_{12}, V_{14}]$ = [2.4928E-06, 1.8614E-05, 3.6817E-05], respectively. Otherwise, GLS estimates were more efficient than OLS. The results for the other non-stochastic designs mimic the same behavior of the linear designs.

Table (3) shows the simulation results of the variances of OLS and GLS estimated variance for standardized normal stochastic design. OLS was more efficient than GLS estimators in estimating β_0 for all sample sizes with Φ = (.2,-.1). For example, when T=50, $[V_0, V_{01}, V_{02}, V_{04}]$= [2.0509E-05, 2.6708E-05, 2.2857E-04, 1.1503E-03].

For estimating the slope, β_1, OLS was nearly as efficient as GLS-AR(2), EGLS-AR(2), and EIGLS-AR(4) estimators for all sample sizes with AR(2) parametrization Φ = (.2,-.1). For example, when T=50, $[V_1, V_{11}, V_{12}, V_{14}]$= [4.5526E-05, 3.9870E-05, 3.8240E-05, 3.6470E-05]. Otherwise, OLS performed poorly. Second, the efficiency of OLS in estimating β_0 was more efficient than EIGLS-AR(1). For example, with AR(2) parametrization Φ = (.2,-.1) for T=50, $[V_0, V_{03}]$= [2.0509E-05,1.6262E-04]. However, the efficiency of OLS in estimating β_1 was nearly as efficient as EIGLS-AR(1), for example, with Φ = (.2,-.1) for T=50, $[V_1, V_{13}]$= [4.5526E-05, 4.0393E-05].

The simulation results based on the ratio of the variance of the estimated variance of OLS to that of GLS of each of the regression coefficients, R_β in (7) are now discussed. Tables (4) and (5) are presented for the linear design.

Table 2: Panel (B) - Variances of OLS and GLS Estimators for Linear Design

Size	Estimator	$(\Phi_1, \Phi_2) = (.2, -.7)$		$(\Phi_1, \Phi_2) = (.2,-.1)$	
		V0	V1	V0	V1
50	VOLS	1.7765E-04	1.8498E-04	1.9062E-05	1.9848E-05
	VGLS AR(2)	3.5205E-06	4.1751E-06	2.5782E-05	2.6844E-05
	VEGLS AR(2)	7.5677E-06	8.6003E-06	2.0664E-04	1.7487E-04
	VEIGLS AR(1)	4.2082E-04	4.1699E-04	1.5224E-04	1.3856E-04
	VEIGLS AR(4)	6.0168E-05	4.8976E-05	8.1543E-04	3.8291E-04
100	VOLS	2.4082E-05	2.4571E-05	2.4432E-06	2.4928E-06
	VGLS AR(2)	4.2622E-07	4.6385E-07	3.2092E-06	3.2743E-06
	VEGLS AR(2)	8.0958E-07	8.6773E-07	1.9476E-05	1.8614E-05
	VEIGLS AR(1)	5.1302E-05	5.1260E-05	1.9368E-05	1.8558E-05
	VEIGLS AR(4)	3.1549E-06	3.1480E-06	4.2741E-05	3.6817E-05
200	VOLS	2.8555E-06	2.8843E-06	3.0692E-07	3.1001E-07
	VGLS AR(2)	5.3668E-08	5.5979E-08	3.9671E-07	4.0070E-07
	VEGLS AR(2)	1.0279E-07	1.0652E-07	2.1704E-06	2.1293E-06
	VEIGLS AR(1)	5.7967E-06	5.7999E-06	2.2070E-06	2.1659E-06
	VEIGLS AR(4)	3.5784E-07	3.6114E-07	3.9794E-06	3.7836E-06

First, when the disturbance term is under parameterization, regardless of the sample size, the selected autoregressive coefficients, and for all the non-stochastic designs, OLS is more efficient than EIGLS-AR(1) in estimating both β_0 and β_1. For example, as shown in Table (4), when Φ = (.8,-.9) for the linear design with T=100, the ratio between V_0 and V_{03} for estimating the intercept, R_{β_0} is about 0.1361, and the ratio between V_1 and V_{13} for estimating the slope, R_{β_1} is about 0.1455. This result indicates that the variance of the OLS estimated variance would be around 0.1361 and 0.1455 times that of EIGLS-AR(1) for estimating the intercept and slope, respectively. This result shows that the variance estimation of EIGLS-AR(1) is at least more than six times higher than the OLS estimator. Moreover, for all cases EIGLS-AR(1) was the least efficient estimator.

Regardless of the example, shown in Table (5) with T=50, the ratio between V_0 and V_{01}, R_{β_0} = 0.7393 and the ratio between V_1 and V_{11}, R_{β_1} = 0.7394. Otherwise, the OLS estimator performed less efficiently than the GLS estimator.

When Φ = (.2,-.1), OLS was more efficient than GLS estimates; EGLS-AR(2), and EIGLS-AR(4), for all sample sizes for all design vectors. For example, as shown in Table (5), for the linear design with sample size T=100, the ratios between the estimated variance of (β_0, β_1) using OLS, EGLS-AR(2) and EIGLS-AR(4) were (0.1254,0.0572) and (0.1339,0.0677), respectively. Otherwise, OLS was less efficient than GLS estimates. The results for the other non-stochastic designs mimic the same behavior of the linear design.

Table 4: Panel (A) - Ratios of OLS and GLS Estimators for Linear Design

Size	Estimator	(Φ_1, Φ_2) = (.2, -.9)		(Φ_1, Φ_2) = (.8,-.9)	
		V0	V1	V0	V1
50	VOLS/VGLS2	1957.3616	1678.7466	460.7095	397.2509
	VOLS/VEGLS2	1312.8309	1146.5926	300.7824	265.2020
	VOLS/VEIGLS1	0.5187	0.5352	0.1134	0.1317
	VOLS/VEIGLS4	181.7205	203.5053	50.1835	55.0002
100	VOLS/VGLS2	2147.5934	1990.3878	518.4137	481.7247
	VOLS/VEGLS2	1683.7773	1570.6721	353.1306	331.2065
	VOLS/VEIGLS1	0.5871	0.5944	0.1361	0.1455
	VOLS/VEIGLS4	303.1820	303.6701	87.5786	87.4111
200	VOLS/VGLS2	1996.0370	1921.9448	638.9684	616.0478
	VOLS/VEGLS2	1631.3452	1574.7931	420.2377	406.7638
	VOLS/VEIGLS1	0.6079	0.6114	0.1463	0.1510
	VOLS/VEIGLS4	368.0923	365.3556	92.6731	92.1673

Table 5: Panel (B) - Ratios of OLS and GLS Estimators for Linear Design

Size	Estimator	$(\Phi_1, \Phi_2) = (.2, -.7)$		$(\Phi_1, \Phi_2) = (.2,-.1)$	
		V0	V1	V0	V1
50	VOLS/VGLS2	50.4616	44.3050	0.7393	0.7394
	VOLS/VEGLS2	23.4750	21.5081	0.0922	0.1135
	VOLS/VEIGLS1	0.4222	0.4436	0.1252	0.1432
	VOLS/VEIGLS4	2.9526	3.7769	0.0234	0.0518
100	VOLS/VGLS2	56.5013	52.9710	0.7613	0.7613
	VOLS/VEGLS2	29.7460	28.3162	0.1254	0.1339
	VOLS/VEIGLS1	0.4694	0.4793	0.1261	0.1343
	VOLS/VEIGLS4	7.6332	7.8053	0.0572	0.0677
200	VOLS/VGLS2	53.2076	51.5253	0.7737	0.7737
	VOLS/VEGLS2	27.7797	27.0774	0.1414	0.1456
	VOLS/VEIGLS1	0.4926	0.4973	0.1391	0.1431
	VOLS/VEIGLS4	7.9800	7.9868	0.0771	0.0819

Table (6) shows the ratio between the variance of OLS estimated variance and the variance of GLS estimates for all sample sizes for the standardized normal design.

First, with Φ = (.2,-.1) and all sample sizes, the ratio between the variance of OLS estimated variance and the variance of GLS estimates; GLS-AR(2), EGLS-AR(2), and EIGLS-AR(4) were significantly smaller than one for estimating an intercept. For example, when T=50, R_{β_0} = (0.7679, 0.0897, 0.0178). (See Table 6.) However, that ratio was slightly larger than the one for estimating the slope. For example, when T=50, R_{β_1} = (1.1419, 1.1905, 1.2483).

Second, regardless of sample size and AR(2) parametrization, the ratio between the variance of OLS estimated variance and the variance of EIGLS-AR(1) was significantly smaller than one for estimating an intercept. For example, with Φ = (.2,-.1) and T=50, R_{β_0} = 0.1261. However, the efficiency of OLS in estimating β_1 was nearly as efficient as EIGLS-AR(1). For example, with Φ = (.2,-.1) and T = 50, R_{β_1} =1.1271.

Conclusion

This study investigated the impact that variance estimators may have on inference based on the OLS estimator. The variance estimation is important because estimates of the variance enter into the usual inferential procedures such as confidence intervals, hypotheses testing, and spectrum estimation, as well as in expressions for the estimated standard error of prediction. The major finding is that, OLS (order zero) estimates outperform first order estimated generalized least squares, EIGLS-AR(1). In particular, the ratio of the variance estimation of the regression coefficients when the disturbance term is under parametrized, i.e. EIGLS-AR(1) has the highest ratio estimation among the other estimators. This indicates that EIGLS-AR(1) can be much less efficient than OLS.

Table 6: Ratios of OLS and GLS Estimators for Standardized Normal Design

		$(\Phi_1, \Phi_2) = (.2, -.7)$		$(\Phi_1, \Phi_2) = (.2,-.1)$	
Size	Estimator	V0	V1	V0	V1
50	VOLS/VGLS2	47.1201	12.0362	0.7679	1.1419
	VOLS/VEGLS2	27.7268	11.5177	0.0897	1.1905
	VOLS/VEIGLS1	0.5036	1.0508	0.1261	1.1271
	VOLS/VEIGLS4	6.1101	11.4395	0.0178	1.2483
100	VOLS/VGLS2	55.4232	13.1856	0.8045	1.1970
	VOLS/VEGLS2	31.4673	12.1070	0.1326	1.1990
	VOLS/VEIGLS1	0.5251	1.0636	0.1252	1.1595
	VOLS/VEIGLS4	8.9087	11.5746	0.0609	1.2367
200	VOLS/VGLS2	55.1432	13.6790	0.7756	1.1285
	VOLS/VEGLS2	31.4317	12.7201	0.1590	1.0819
	VOLS/VEIGLS1	0.5338	1.0553	0.1524	1.0836
	VOLS/VEIGLS4	10.0527	11.8398	0.0756	1.1029

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